Week 14 Worksheet Tuesday - Integration

Instructions. It's time for integration! Follow the instructions given by your TA. You are not expected to finish all the problems. :)

Group 1:

1. $\int 2x^4 + \frac{5}{x^3} - \frac{1}{2x} + 4\sqrt{x^3} - 8\, dx$ $=$ $2\frac{x^5}{5} + 5\frac{x^{-2}}{-2} - \frac{1}{2}\ln|x| + 4\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 8x + C$

   ($5x^{-3}$, $4x^{\frac{3}{2}}$)

2. $\int (x^2 - 2)(x^2 + 1)\, dx = \int x^4 - x^2 - 2\, dx = \frac{x^5}{5} - \frac{x^3}{3} - 2x + C$

3. $\int \frac{y^2 - 3y^4}{y^2}\, dy = \int 1 - 3y^2\, dy = y - 3\frac{y^3}{3} + C$

4. $\int_0^{\frac{\pi}{4}} 2\sin x - \sec^2 x\, dx = -2\cos x - \tan x \Big|_{x=0}^{x=\frac{\pi}{4}} = -2\cos\frac{\pi}{4} - \tan\frac{\pi}{4} + 2\cos 0 + \tan 0 = -\sqrt{2} + 1$

Group 2:

1. $\int (2x - 3)^{15}\, dx$

2. $\int x^2 \sin(x^3)\, dx$

3. $\int \frac{1}{2x^2} - \sec^2(\pi x)\, dx$

4. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 x \cos x\, dx$

5. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

6. $\int x^4 (3 - 5x^5)^{\frac{1}{3}}\, dx$

7. $\int \frac{2-x}{\sqrt{2x^2 - 8x + 1}}\, dx$

8. $\int \frac{1}{x\ln x}\, dx$

9. $\int x^2\sqrt{x - 1}\, dx$ (a bit tricky!)

10. $\int \frac{dx}{\sqrt{1-x^2}\,\arcsin x}$

#3. $? = \int \frac{1}{2} x^{-2}\, dx - \int \sec^2(\pi x)\, dx$ $u = \pi x$

$= \frac{1}{2}\frac{x^{-1}}{-1} - \int \sec^2 u \frac{du}{\pi}$ $du = \pi\, dx$

$= -\frac{1}{2} x^{-1} - \frac{1}{\pi}\tan(u) + C$

$= -\frac{1}{2} x^{-1} - \frac{1}{\pi}\tan(\pi x) + C$

#4. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x)^2 \cos x\, dx$ $u = \sin x$

$= \int_{u=\sin\frac{\pi}{4}}^{u=\sin\frac{\pi}{3}} u^2\, du$ $du = \cos x\, dx$

$= \frac{u^3}{3} \Big|_{u=\frac{\sqrt{2}}{2}}^{u=\frac{\sqrt{3}}{2}}$

$= \frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)^3 - \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3$

#5 $u = \sqrt{x}$ $du = \frac{1}{2}\frac{1}{\sqrt{x}}\, dx$

$? = \int \sin u \cdot 2\, du$

$= -2\cos u + C$

$= -2\cos\sqrt{x} + C$

Group 2:

#1 $u = 2x - 3$ $du = 2\, dx$

$? = \int u^{15}\frac{du}{2}$

$= \frac{1}{2}\int u^{15} du = \frac{1}{2}\frac{u^{16}}{16} + C$

$= \frac{1}{2}\frac{(2x-3)^{16}}{16} + C$

#2 $u = x^3$ $du = 3x^2\, dx$

$? = \int \sin u \frac{du}{3}$

$= -\frac{1}{3}\cos u + C$

$= -\frac{1}{3}\cos(x^3) + C$

#6. $u = 3-5x^5$

$du = -25x^4 dx$

$? = \int u^{\frac{1}{3}} \dfrac{du}{-25}$

$= -\dfrac{1}{25} \dfrac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$

$= -\dfrac{1}{25} \dfrac{3(3-5x^5)^{\frac{4}{3}}}{4} + C$

#7 $u = 2x^2-8x+1$

$du = 4x-8 \ dx \qquad \Rightarrow dx = \dfrac{du}{4x-8}$

$? = \int \dfrac{2-x}{\sqrt{u}} \dfrac{du}{4x-8}$

$= \int \dfrac{-(x-2)}{\sqrt{u}} \dfrac{du}{4(x-2)}$

$= -\dfrac{1}{4} \int u^{-\frac{1}{2}} du$

$= -\dfrac{1}{4} \dfrac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= -\dfrac{1}{2} (2x^2-8x+1)^{\frac{1}{2}} + C$

#8 $u = \ln x$

$du = \frac{1}{x} dx$

$? = \int \dfrac{1}{u} du$

$= \ln|u| + C$

$= \ln|\ln x| + C$

#9. $u = x-1 \qquad \Rightarrow u+1 = x$

$du = dx$

$? = \int x^2 u^{\frac{1}{2}} du$

$= \int (u+1)^2 u^{\frac{1}{2}} du$

$= \int (u^2+2u+1) u^{\frac{1}{2}} du$

$= \int u^{2.5} + 2u^{1.5} + u^{0.5} du$

$= \dfrac{u^{3.5}}{3.5} + 2\dfrac{u^{2.5}}{2.5} + \dfrac{u^{1.5}}{1.5} + C$

$= \dfrac{(x-1)^{3.5}}{3.5} + 2\dfrac{(x-1)^{2.5}}{2.5} + \dfrac{(x-1)^{1.5}}{1.5} + C$

#10 $u = \arcsin x$

$du = \dfrac{1}{\sqrt{1-x^2}} dx$

$? = \int \dfrac{1}{u} du$

$= \ln|u| + C$

$= \ln|\arcsin x| + C$